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Comparative analysis of interest rate term structures in the Solvency II environment

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Abstract

Purpose – Solvency-II is the current regulatory framework of insurance companies in the European Union. Under this standard, European Insurance and Occupational Pension Authority (EIOPA), as a regulatory board, has established that the Smith–Wilson (SW) model can be used as the model to estimate interest rate curve. This paper aims to analyze whether this model adjusts to the market curve better than Nelson–Siegel (NS) and whether the values set for the parameters are adequate.

Design/methodology/approach – This empirical study analyzes whether the SW interest rate curve shows lower root mean squared errors than the NS curve for a sample of daily prices of Spanish Government bonds between 2014 and 2019.

Findings – The results indicate that NS adjusts the market data better, the parameters recommended by the EIOPA correspond to the maximum values observed in the sample period and the current recommended curve for insurance companies underestimates company operations.

Originality/value – This paper verifies that the criterion of the last liquid point does not allow for selecting an optimal sample to adjust the curve and criteria based on prices without arbitrage opportunities are more appropriate.

Keywords Last liquid point, Nelson–Siegel, Smith–Wilson, Solvency-II, Ultimate forward rate

Paper type Technical paper

1. Introduction

The current regulatory norms in the European Union (EU) for insurance companies, known as Solvency-II, require estimating the term structure of interest rates for the valuation of operations. This curve has become a key factor for insurers' financial management.

The European Insurance and Occupational Pension Authority (EIOPA), as the supervisory board of insurance companies and pension plans in the EU, has developed several technical documents (EIOPA (2017) and EIOPA (2018)) to help insurance companies estimate interest rate curves. In this paper, we plan to study a few of the key issues mentioned in those technical documents empirically.

First, we examine the suggested model. The EIOPA (2017) recommends the Smith–Wilson (SW) model among the long-term convergence models. However, is this the best option? To verify, we have compared the empirical results of applying this model to those in which in the long term the model converges to an interest rate, the Nelson–Siegel (NS) model, one frequently used in the financial sector.

Second, the EIOPA (2018) selects the sample of bonds to be used when estimating the curve, applying the so-called last liquid point (LLP) criterion, i.e. it considers that only liquid bonds that do not exceed a certain maturity should be used. It recommends 20 years

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for bonds issued by Euro zone countries. However, for the rest of European and non-European currencies, the EIOPA determines that the LLP varies between 8 and 50 years, depending on the peculiarities of each currency and as such of each country. Should bonds with a maturity of more than 20 years not be included in the curve estimate sample for EU countries?

The EIOPA also sets values for the SW model parameters. The long-term rate, or ultimate forward rate (UFR), should be equal to 4.2% (obtained by adding the expected real interest rate of 2.2% and the inflation target of 2%). It also indicates that the rate of reversion at this rate is usually around 10%. A question naturally arises: are these values adequate in any scenario? According to Jørgensen (2018), the parameters set by the EIOPA considerably increase the interest rate curve and artificially move it away from its true position, as set by the market.

In contrast to the above, European banking (Nymand-Andersen, 2018) uses parametric models from the NS family. This different approach to modeling the interest rate curve in the EU give rise to our main goal and therefore the main contribution of this empirical research is to determine which of the two approaches best adjusts the market prices of the bonds. Given that the regulation affects EU countries, the effects of both models are studied in a country that is part of the EU, Spain, for two fundamental reasons: first, there are major insurance companies (MAPFRE, MUTUA, etc.) and banks in this country (Santander, BBVA, etc.), and also by considering only one country we avoid including country risk in the modeling, which would be necessary if we added other countries with different risk premiums and the same currency (Euro), because this would make it harder to interpret the results obtained. Our empirical analysis has been carried out with all Spanish Government bonds traded daily from 2014 to 2019.

This paper is organized as follows: Section 2 presents a review of the literature regarding the interest rate curves of bonds that we intend to compare. Section 3 shows the methodology applied to compare the results of the estimated curves. Section 4 describes the sample used in the empirical study. Section 5 contains the results of the study. Finally, in Section 6, we state the main conclusions of the comparative analysis.

2. Literature review

There are, in the literature, different methods for estimating the interest rates term structure with convergence at a long-term rate. Among those methods, in this paper we will focus on the SW model proposed by the EIOPA and the NS model, which is used by a large number of central banks, economic agents and researchers.

In the SW model (Smith and Wilson, 2001) the current price of an iBond is given by:

$$P_i = \sum_{j=1}^{J} c_{i,j} \cdot \left[\exp\left(-UFR \cdot t_j \right) + \sum_{k=1}^{K} z_k \cdot \left(\sum_{j=1}^{J} c_{k,j} \cdot W_{\tau(k),\tau(j)} \right) \right] + \epsilon_i \tag{1}$$

where P_t is coupon bond market price with maturity in T; c_j is each bond cash flow with cash payment dates $\tau(j)$; UFR is long-term discount rate at which the curve converges from the LLP; z_k represents the parameters to be estimated for each of the K bonds of sample, which compose the estimation sample and $W_{\tau(k),\tau(j)}$ is the Wilson function defined as:

$$\begin{aligned} W_{t,\tau(j)} &= \exp[-UFR \cdot (t + \tau(j))] \\ &\quad \cdot \{\alpha \cdot min(t, \tau(j)) - \exp[-\alpha \cdot max(t, \tau(j))] \cdot \sinh[\alpha \cdot min(t, \tau(j))]\} \end{aligned} \tag{2}$$

where α represents the speed of convergence of interest rates to *UFR*. The EIOPA sets minimum value $\alpha = 0.1$. In our case, given the lack of linearity in the parameters of 2, we estimate the parameters through a non-linear least squares optimization routine, using the *BFGS* optimizer.

However, EIOPA (2017) remarks that there is more than one method to extrapolate interest rates. In addition, there is a lack of consensus about which one is the best, because all the methods have different pros and cons. In recent years, many authors have tried to explain whether the method we use in this paper is appropriate or not. Lageras and Lindholm (2016) find a case in which, satisfying the convergence criteria, the SW discount curve takes negative values (i.e. for values belonging to the logical parameter range ($\alpha \geq 0$), thus, they find singularities lacking financial logic.

Christensen *et al.* (2019) analyze the adequacy of the SW method and although their work is based on observable data, it does not incorporate market expectations of the future yield curve dynamics as reflected in the traded prices of government bonds.

EIOPA (2018), despite recommending the SW model, also makes explicit mention of the NS method as a simple econometric method for estimating the curve.

Nelson and Siegel (1987) argues that the forward rate curve must be build assuming that the forward rates converge asymptotically at a certain level, such as SW. The NS model aims to describe the yield curve taking into account three factors: level, slope and curvature. We use an econometric specification of NS model for estimating it:

$$y_i = \beta_0 + \beta_1 \cdot X_{1,i} + \beta_2 \cdot X_{2,i} + u_{i,t} \tag{3}$$

where, at any time, estimation sample is composed by N bonds ($i=1,\ldots,N$) and y_i is the observed yield of bond i, which is estimated from market price and its characteristics (maturity and coupon). Finally, the X regressors in Expression 3 are defined as:

$$\begin{aligned} X_{1,i} &= \frac{1 - \exp\left(\frac{-t_i}{\tau}\right)}{\frac{t_i}{\tau}} \\ X_{2,i} &= X_{1,i} - \exp\left(\frac{-t_i}{\tau}\right) \end{aligned} \tag{4}$$

where t_i is the time remaining until maturity of bond i in each estimate date. The parameters to be estimated are as follows:

- β_0, the model's constant (interpreted as an estimation of the long-term convergence interest rate), equivalent to *UFR* in SW.
- β_1 is the slope of the curve. If it is negative, the curve is decreasing and if it is positive, it is increasing. It also represents the short-term factor of the curve.
- β_2 is the curvature of the interest rate curve. If it is positive, it is concave, and if it is negative, it is convex. At the same time, it indicates the medium-term factor of the curve.
- τ is the inverse of the speed with which forward rates converge to long-term rates. For a lower (always positive) value, the convergence rate is higher and is similar to the parameter in the SW model. Additionally, it represents the form of the function and shows the maturity by which the medium-term factor takes the maximum value.

Note, from the Expression 4, that by definition, there is multicollinearity between the regressors. We apply the methodology of Gimeno and Nave (2019), Gauthier and Simonato (2012), Annaert *et al.* (2013) and González-Sánchez (2018) to estimate Expression 4, avoiding this drawback.

The original NS model has been treated profusely in the literature by adding different transformations. Svensson (1994) adds a fourth term that allows a second curvature; however, this increases the problems of multicollinearity. Diebold and Li (2006) and Diebold and Rudebusch (2013) introduce dynamic factors in the model. Coroneo *et al.* (2011) analyze the model under the assumption of non-arbitrage.

For the purposes of our study, the main problem presented by the NS method is the estimation of the model. There are two main approaches in the parameter estimation techniques of the model using time series models (the so-called dynamic approach as per Diebold and Li (2006) and Koopman *et al.* (2010) and the cross-sectional approach (De Pooter, 2007). It is important to note that the time series approach has the disadvantage of prior fixing the temporal behavior of the parameters beforehand.

3. Research design

Along with Berenguer *et al.* (2013), our approach to compare the estimation models of interest rate curves, as well as the hypotheses related to *UFR* and *LLP*, consists of measuring the estimation error, specifically the square root of the mean square error (*RMSE*). We estimate both models each day (t) of sample period, supposing that full sample consists of T days plus a maximum of N bonds as observations each day. P^M is the market price of the bond and P^{SW} and P^{NS} are the estimated prices by SW and NS models, respectively. So, the $RSME_{full}$ is the *RMSE* for the full sample and it is calculated as:

$$RMSE^j_{full} = \sqrt{\frac{1}{N \cdot T} \cdot \sum_{i=1}^{N} \left(P^M_{i,t} - P^j_{i,t} \right)^2} \tag{5}$$

where j stands for SW or NS, i.e. one of the two estimation methods.

We repeat the *RMSE* estimate N-times but removing a bond of the sample each time. Then, we get the estimate error of eliminating a bond k from the sample as follows:

$$\forall i \neq k \qquad RMSE^j_{1,k} = \sqrt{\frac{1}{(N-1) \cdot T} \cdot \sum_{i=1}^{N-1} \left(P^M_{i,t} - P^j_{i,t} \right)^2} \tag{6}$$

We also estimate the error with respect to the price of the bond excluded as:

$$RMSE^j_{2,k} = \sqrt{\frac{1}{T} \cdot \sum_{t=1}^{T} \left(P^M_{k,t} - P^j_{k,t} \right)^2} \tag{7}$$

We assume that, as a consequence of excluding a bond from the sample, if the estimate error increases considerably ($RMSE^j_{1,k}$), this would indicate that the price of this bond includes essential information to price the rest of the bonds. At the same time, to correct possible endogeneity problems, we also consider the pricing error of the excluded bond ($RMSE^j_{2,k}$). This way, a bond is included in the best sample of estimation, provided that:

$$\frac{RMSE^j_{1,k} + RMSE^j_{2,k}}{RMSE^j_{full}} \geq 1 + \delta \tag{8}$$

where δ is the tolerance level of bias (for our purpose, δ is set to 1%). Note that a bond excluded from the best sample to estimate the interest rate curve, and therefore included in the worst sample, means that total error owing to its exclusion (error type-1 plus type-2) is less than the error of including it (*full*).

However, this methodology depends on the estimation model and, additionally, the exclusion or inclusion of a bond affects the entire sample period. For example, including a bond in the estimate sample might be optimal only at some times and another is better to exclude it. We propose another method of selecting the optimal sample, which we call high accuracy and free-arbitrage sample, because it consists of estimating at each date t, the replica portfolio of each bond k and then checking whether the market value of the bond (P_k^M) differs from the value of the replica portfolio (P_k^{arb}) with equal yield (return) and duration (risk) more than 0.01 Euros. To estimate the replica portfolio of k bond, we solve the following equation:

$$\begin{pmatrix} y_1 & \cdots & y_N \\ D_1 & \cdots & D_N \\ 1 & \cdots & 1 \end{pmatrix} \bullet \begin{pmatrix} w_1 \\ \vdots \\ w_N \end{pmatrix} = \begin{pmatrix} y_k \\ D_k \\ 1 \end{pmatrix} \tag{9}$$

where y is the yield of each bond, D is Macaulay duration and w are the weights of each bond in replicating portfolio. Equation (9) could be expressed in a matrix form such as $\mathbf{A}\bullet\mathbf{w} = \mathbf{b}$, and the solution is $\mathbf{w} = \mathbf{A}^{-1}\bullet\mathbf{b}$. The problem is that the matrix $\boldsymbol{A}_{3x(N-1)}$ is not squared, so we estimate its inverse by singular-value decomposition. For each date t and, once we have solved Expression 9, the expected value of bond k is as follows:

$$\begin{pmatrix} P_1^M & \cdots & P_N^M \end{pmatrix} \bullet \begin{pmatrix} w_1 \\ \vdots \\ w_N \end{pmatrix} = P_k^{arb} \tag{10}$$

Then, if $\left| P_{k,t}^M - P_{k,t}^{arb} \right| \geq 0.01$, the bond k in date t is excluded of optimal sample to estimate interest rate curve for this date, because if included, the tolerance level allowed is exceeded.

In this way, we obtain three optimal sub-samples, two depending on the *RMSE* criteria for each model and another one based on the arbitrage criteria and exogenous to the curve model. Consequently, we compare the estimate errors and the resulting parameters for each model and sub-sample, which allows us to establish that the binomial of selection criteria and curve model is ideal. From the optimal sub-sample and model, we also check whether a 4.2% UFR and an the minimum value as 10% long rate of reversion speed adjusted to the sample reality.

Finally, we study if the maturity is an ideal criterion to select the bonds that should be part of the estimate sample. To do so, we use two estimates.

For each model, we define the variable z_i as a dummy variable that takes the value 1 if the bond i is part of the optimal sample and zero otherwise. Then, we estimate a logit model for each curve model (j=*SW,NS*) such that:

$$Prob\left(z_{i,j} = 1\right) = \frac{1}{\exp\left[-\left(\alpha_{0,j} + \alpha_{1,j}\cdot M_i + \alpha_{2,j}\cdot c_i + 1\right)\right]} \tag{11}$$

where M and c are the maturity and the coupon of the bond, respectively. According to EIOPA, the expected value of maturity effect is $\alpha_1 < 0$, because the higher the maturity the lower the liquidity and then, a lower probability of bond being included in the optimal sub-sample for estimating interest rate curve.

For the optimal sub-sample applying the arbitrage criteria, we define $f_i = \frac{n_i}{T_i}$ as exclusion frequency of bond i, estimated as the ratio of the number of days that the bond is not part of the sample (n_i) and the total number of days that the bond is quoted during the sample period ($T_i \geq T$), where T is the total sample period. Then, we estimate the model by least squares:

$$f_i = \gamma_0 + \gamma_1 \cdot M_i + \gamma_2 \cdot c_i + \xi_i \qquad \xi_i \sim i.i.d.(0, \sigma^2) \tag{12}$$

If the EIOPA hypothesis is true, then higher maturity involves higher exclusion frequency and we expect that $\gamma_1 > 0$, i.e. the exclusion frequency increases γ_1% as the maturity increases by one year.

4. Data

To achieve our objective, we use daily data from Spanish Government (sovereign) bonds, which were gathered from the Bloomberg database. Our dataset spans the period from January 1, 2014, to June 30, 2019, with daily frequency. Table 1 shows the characteristics of the 72 bonds traded in the secondary public debt market in the sample period including the number of days in that period. These are all the bonds that were traded during the sample period:

Table 2 shows a summary of the sample obtained:

Regarding our objective, two issues are worth mentioning: first, to reach 20 years as the LLP (as in the EIOPA proposal), 0.76 times the standard deviation must be added to the mean maturity, which represents 78% (percentile) of the sample under the hypothesis of normal behavior. With regard to the long-term rate of 4.2% and its relation to the coupon, we have to add 0.65 times the standard deviation to the mean coupon, so that it represents 74% of the sample under the normal distribution behavior hypothesis.

5. Results and discussion

First, we show the results of the *RMSE* when a single bond has been removed from the sample and both models are re-estimated (NS and SW). We also include the results under the arbitrage selection criterion, which does not consider any curve model to avoid model risk problems because of the selection of the most liquid bonds to estimate the curve in each day of sample period. Table 3 shows the results for every bond.

Analyzing the results of Table 3, we find several issues relevant to our objective.

First, we observe that the NS model always shows a smaller *RMSE* than SW, both for the bond excluded and for the rest of the bonds included in the daily estimate of the curve. This means that the sample chosen as the best (*Best*) for each model does not include the same bonds, and the same is true with the worst (*Worst*). From this, we deduce that the SW model is not the best to adjust the market curve and, therefore, sometimes entails a high estimate error. We also observe that the selection of bonds to estimate the optimal curve must be made independently of the estimate model; otherwise, we should take into account the model risk.

Second, we should point out that the exclusion of bonds cannot be performed in an absolute way, i.e. any bond can be excluded from the optimal sample to estimate the curve, for some dates, logically attending to market prices and the information contained in them.

To support these conclusions, Table 4 shows the *RMSE* for each model based on the sample used for the estimate.

Table 5 shows quartiles of estimated parameters from each sub-sample of the NS model.

Table 1.
Sample of Spain Government bonds

Designation	Reference	Maturity	Issue date	Coupon (%)	Coupon per year	Remaining days
B-1	SPGB 4.25 January 31, 2014, Government	January 31, 2014	October 17, 2008	4.25	1	30
B-2	SPGB 3.4 April 30, 2014 Government	April 30, 2014	April 12, 2011	3.40	1	119
B-3	SPGB 4.75 July 30, 2014 Government	July 30, 2014	December 07, 1998	4.75	1	210
B-4	SPGB 3.3 October 31, 2014 Government	October 31, 2014	July 07, 2009	3.30	1	303
B-5	SPGB 4.4 January 31, 2015 Government	January 31, 2015	June 28, 2004	4.40	1	395
B-6	SPGB 2.75 March 31, 2015 Government	March 31, 2015	January 15, 2013	2.75	1	454
B-7	SPGB 3 April 30, 2015 Government	April 30, 2015	March 09, 2010	3.00	1	484
B-8	SPGB 4 July 30, 2015 Government	July 30, 2015	January 17, 2012	4.00	1	575
B-9	SPGB 3.75 October 31, 2015 Government	October 31, 2015	September 25, 2012	3.75	1	668
B-10	SPGB 3.15 January 31, 2016 Government	January 31, 2016	September 20, 2005	3.15	1	760
B-11	SPGB 3.25 April 30, 2016 Government	April 30, 2016	November 09, 2010	3.25	1	850
B-12	SPGB 3.3 July 30, 2016 Government	July 30, 2016	April 09, 2013	3.30	1	941
B-13	SPGB 4.25 October 31, 2016 Government	October 31, 2016	September 06, 2011	4.25	1	1,034
B-14	SPGB 3.8 January 31, 2017 Government	January 31, 2017	October 18, 2006	3.80	1	1,126
B-15	SPGB 2.1 April 30, 2017 Government	April 30, 2017	November 26, 2013	2.10	1	1,215
B-16	SPGB 5.5 July 30, 2017 Government	July 30, 2017	March 11, 2002	5.50	1	1,306
B-17	SPGB 4.75 September 30, 2017 Government	September 30, 2017	November 29, 2012	4.75	1	1,368
B-18	SPGB 0.5 October 31, 2017 Government	October 31, 2017	September 23, 2014	0.50	1	1,134
B-19	SPGB 4.5 January 31, 2018 Government	January 31, 2018	November 13, 2012	4.50	1	1,491
B-20	SPGB 0.25 April 30, 2018 Government	April 30, 2018	May 26, 2015	0.25	1	1,070
B-21	SPGB 4.1 July 30, 2018 Government	July 30, 2018	February 19, 2008	4.10	1	1,671
B-22	SPGB 3.75 October 31, 2018 Government	October 31, 2018	July 09, 2013	3.75	1	1,764
B-23	SPGB 2.45 October 31, 2018 Government	October 31, 2018	January 30, 2014	2.45	1	1,735
B-24	SPGB 0.25 January 31, 2019 Government	January 31, 2019	January 26, 2016	0.25	1	1,101
B-25	SPGB 2.75 April 30, 2019 Government	April 30, 2019	January 14, 2014	2.75	1	1,932
B-26	SPGB 4.6 July 30, 2019 Government	July 30, 2019	February 10, 2009	4.60	1	2,004
B-27	SPGB 4.3 October 31, 2019 Government	October 31, 2019	June 02, 2009	4.30	1	2,004
B-28	SPGB 1.4 January 31, 2020 Government	January 31, 2020	July 08, 2014	1.40	1	1,816
B-29	SPGB 4 April 30, 2020 Government	April 30, 2020	January 20, 2010	4.00	1	2,004
B-30	SPGB 1.15 July 30, 2020 Government	July 30, 2020	June 16, 2015	1.15	1	1,473
B-31	SPGB 4.85 October 31, 2020 Government	October 31, 2020	July 13, 2010	4.85	1	2,004
B-32	SPGB 0.05 January 31, 2021 Government	January 31, 2021	June 06, 2017	0.05	1	752

(*continued*)

Designation	Reference	Maturity	Issue date	Coupon (%)	Coupon per year	Remaining days
B-33	SPGB 5.5 April 30, 2021 Government	April 30, 2021	January 24, 2011	5.50	1	2,004
B-34	SPGB 0.75 July 30, 2021 Government	July 30, −2021	March 08, 2016	0.75	1	1,207
B-35	SPGB 0.05 October 31, 2021 Government	October 31, 2021	October 09, 2018	0.05	1	262
B-36	SPGB 3.82 January 31, 2022 Government	January 31, 2022	January 30, 2014	3.82	1	1,975
B-37	SPGB 5.85 January 31, 2022 Government	January 31, 2022	November 22, 2011	5.85	1	2,004
B-38	SPGB 0.4 April 30, 2022 Government	April 30, 2022	January 24, 2017	0.40	1	885
B-39	SPGB 0.45 October 31, 2022 Government	October 31, 2022	October 10, 2017	0.45	1	626
B-40	SPGB 5.4 January 31, 2023 Government	January 31, 2023	January 29, 2013	5.40	1	2,004
B-41	SPGB 0.35 July 30, 2023 Government	July 30, 2023	May 22, 2018	0.35	1	402
B-42	SPGB 4.4 October 31, 2023 Government	October 31, 2023	May 21, 2013	4.40	1	2,004
B-43	SPGB 4.8 January 31, 2024 Government	January 31, 2024	September 16, 2008	4.80	1	2,004
B-44	SPGB 3.8 April 30, 2024 Government	April 30, 2024	January 29, 2014	3.80	1	1,976
B-45	SPGB 0.25 July 30, 2024 Government	July 30, 2024	April 16, 2019	0.25	1	73
B-46	SPGB 2.75 October 31, 2024 Government	October 31, 2024	June 20, 2014	2.75	1	1,834
B-47	SPGB 1.6 April 30, 2025 Government	April 30, 2025	January 27, 2015	1.60	1	1,613
B-48	SPGB 4.65 July 30, 2025 Government	July 30, 2025	February 24, 2010	4.65	1	2,004
B-49	SPGB 2.15 October 31, 2025 Government	October 31, 2025	June 09, 2015	2.15	1	1,480
B-50	SPGB 1.95 April 30, 2026 Government	April 30, 2026	January 19, 2016	1.95	1	1,256
B-51	SPGB 5.9 July 30, 2026 Government	July 30, −2026	March 15, 2011	5.90	1	2,004
B-52	SPGB 1.3 October 31, 2026 Government	October 31, 2026	July 26, 2016	1.30	1	1,067
B-53	SPGB 1.5 April 30, 2027 Government	April 30, 2027	January 31, 2017	1.50	1	878
B-54	SPGB 1.45 October 31, 2027 Government	October 31, 2027	July 04, 2017	1.45	1	724
B-55	SPGB 1.4 April 30, 2028 Government	April 30, 2028	January 30, 2018	1.40	1	514
B-56	SPGB 1.4 July 30, 2028 Government	July 30, 2028	July 03, 2018	1.40	1	360
B-57	SPGB 5.15 October 31, 2028 Government	October 31, 2028	July 16, 2013	5.15	1	2,004
B-58	SPGB 6 January 31, 2029 Government	January 31, 2029	January 15, 1998	6.00	1	2,004
B-59	SPGB 1.45 April 30, 2029 Government	April 30, 2029	January 29, 2019	1.45	1	150
B-60	SPGB 0.6 October 31, 2029 Government	October 31, 2029	June 19, 2019	0.60	1	9
B-61	SPGB 1.95 July 30, 2030 Government	July 30, 2030	March 04, 2015	1.95	1	1,577
B-62	SPGB 5.75 July 30, 2032 Government	July 30, 2032	January 23, 2001	5.75	1	2,004
B-63	SPGB 2.35 July 30, 2033 Government	July 30, 2033	March 01, 2017	2.35	1	849
B-64	SPGB 1.85 July 30, 2035 Government	July 30, 2035	March 05, 2019	1.85	1	115

(*continued*)

Table 1.

Table 1.

Designation	Reference	Maturity	Issue date	Coupon (%)	Coupon per year	Remaining days
B-65	SPGB 4.2 January 31, 2037 Government	January 31, 2037	January 17, 2005	4.20	1	2,004
B-66	SPGB 4.9 July 30, 2040 Government	July 30, 2040	June 20, 2007	4.90	1	2,004
B-67	SPGB 4.7 July 30, 2041 Government	July 30, 2041	September 28, 2009	4.70	1	2,004
B-68	SPGB 5.15 October 31, 2044 Government	October 31, 2044	October 16, 2013	5.15	1	2,004
B-69	SPGB 2.9 October 31, 2046 Government	October 31, 2046	March 15, 2016	2.90	1	1,200
B-70	SPGB 2.7 October 31, 2048 Government	October 31, 2048	February 27, 2018	2.70	1	486
B-71	SPGB 4 October 31, 2064 Government	October 31, 2064	September 08, 2014	4.00	1	1,754
B-72	SPGB 3.45 July 30, 2066 Government	July 30, 2066	May 18, 2016	3.45	1	1,136

In the results of Table 5, we observe that the parameters obtained are very different from one sample to the next. Likewise, the inverse of τ, or reversion speed to the long-term rate, never matches the 10%, the value proposed by EIOPA. Note that β_0 is the long-term rate of convergence and only in the case of the maximum value, the results are close to the $URF = 4.2\%$, proposed by EIOPA. We also observe that the parameters showing the effect of the medium term (β_1) and short term (β_2) are very diverse, so it is evident that the increased flexibility of the NS model owe the SW means that NS fits the market data better and this justifies the lower estimate error discussed above. Finally, note that the minimum (*min bonds*) and maximum (*max bonds*) number of bonds used in the estimate for a single day also differs from one sample to another, which undoubtedly influences the estimation error.

In the same way as for the NS model, Table 6 shows the quartiles of the parameters estimated for the SW model.

From the results of Table 6, we verify that only when the estimates are close to the maximum, the parameters proposed by EIOPA ($\alpha = 0.1$ and $URF = 4.2\%$) are consistent with those estimated from the market data. As with the NS model, we observe a wide range of parameters and sample sizes depending on the criterion to choose the bonds.

Finally, although SW model has a lower complexity of estimation than NS because of the smaller number of parameters used in the model and this could entail a lower computational cost, this is not the case. The two-stage NS estimate (using a DELL Precision M6500 mobile workstation computer with RAM 32GB, Intel Core i7 processor and two HDD hard drives of 465 GB each) takes about 1 min and 15 s for each NS estimation, whereas the SW model consumes approximately 5 min and 37 s.

Table 7 shows the relationship between the maturity of the bond and their coupons with respect to their exclusion in the optimal samples (*Best*). For the selection made according to the *RMSE* of each model (Panel A for NS and Panel B for SW), in which the response should be excluded or not, and as such is binary, we estimate a logit model as indicated above. However, we use a multiple linear regression model (Panel C) for the selection made, which is based on arbitrage, because the answer is the number of days that a bonus is excluded from the optimal sample.

From results in Table 7, we observe that only the NS model and the selection by arbitrage exhibit a relationship with the bond maturity, i.e. the maturity increases the probability excluding the bond in the NS optimal sample (positive and significant parameter of 0.0866). The results show that the number of exclusion days for a bond from the optimal sample by arbitrage are five days per maturity year (positive and significant parameter of 4.78, approximately five days excluded per year of maturity). However, the maturity is not significant for the SW model. For every model and sample, no statistical significance is found

Item	Value
Observations	60,455
Number of bonds	72
Market mean price	115.23
Standard deviation of price	14.25
Mean maturity in years	12.11
Standar deviation of maturity	10.42
Mean coupon	3.08%
Standard deviation of coupon	1.71%

Table 2.
Summary of sample

Table 3.
Results of selection bonds for optimal sample

Denomination	RMSE NS Bond excluded	Rest of bonds	NS best	RMSE SW Bond excluded	Rest of bonds	SW best	Arbitrage selection Exclusions	Exclusion days (%)
B-1	0.01738	7.25799	No	0.64729	14.73915	No	0	0.00
B-2	0.25623	7.26046	No	0.50301	15.51503	No	0	0.00
B-3	0.15427	7.26419	No	0.81026	15.16128	No	0	0.00
B-4	0.12486	7.26805	No	0.68306	14.73646	No	0	0.00
B-5	0.12958	7.27195	No	0.32057	15.979	Yes	0	0.00
B-6	0.15419	7.27509	No	0.91503	15.16599	No	0	0.00
B-7	0.18756	7.27695	No	0.66121	14.72841	No	0	0.00
B-8	0.12125	7.27951	No	0.99435	14.9052	No	0	0.00
B-9	0.24998	7.28341	No	0.45553	15.03935	No	0	0.00
B-10	0.14151	7.28732	No	0.55419	14.65401	No	0	0.00
B-11	0.09893	7.29118	No	0.43913	15.32776	No	0	0.00
B-12	0.1597	7.29505	No	0.45752	15.39715	No	22	2.34
B-13	0.25957	7.29893	No	0.07521	14.50184	No	46	4.45
B-14	0.19622	7.30286	No	0.81206	14.70488	No	62	5.51
B-15	0.0799	7.30676	No	0.4775	15.85031	Yes	82	6.75
B-16	0.22052	7.31064	No	0.08903	14.83698	No	108	8.27
B-17	0.22071	7.30805	No	0.64621	15.23149	No	131	9.58
B-18	0.03201	7.30021	No	0.87468	14.87912	No	147	12.96
B-19	0.32094	7.68713	Yes	0.5759	16.06506	Yes	162	10.87
B-20	0.22777	7.35362	No	0.84402	15.64236	No	175	16.36
B-21	0.30213	7.32287	No	0.64952	14.80871	No	220	13.17
B-22	0.31554	7.33117	No	0.19484	15.5517	No	228	12.93
B-23	0.30571	9.41256	Yes	0.16191	15.94159	Yes	235	13.54
B-24	0.30777	7.88557	No	0.97879	15.94147	Yes	244	22.16
B-25	0.40528	20.84951	Yes	0.04337	14.6913	No	300	15.53
B-26	0.45367	11.85925	Yes	0.89175	15.50084	No	298	14.87
B-27	0.44831	8.17652	Yes	0.2074	15.56238	No	322	16.07
B-28	0.38371	6.57636	No	0.06491	15.11415	No	274	15.09
B-29	0.41051	5.94328	No	0.01002	15.06968	No	317	15.82
B-30	0.30027	5.77358	No	0.66865	15.56232	No	300	20.37
B-31	0.3132	5.85976	No	0.46402	15.58538	No	320	15.97

(*continued*)

Denomination	RMSE NS Bond excluded	Rest of bonds	NS best	RMSE SW Bond excluded	Rest of bonds	SW best	Arbitrage selection Exclusions	Exclusion days (%)
B-32	0.2411	6.07617	No	0.21529	15.87496	Yes	205	27.26
B-33	0.31206	6.37661	No	0.01226	15.80997	Yes	296	14.77
B-34	0.20212	6.69945	No	0.30522	15.0892	No	291	24.11
B-35	0.08094	6.97818	No	0.47897	15.7315	Yes	63	24.05
B-36	0.37468	7.27051	No	0.4166	15.70835	Yes	315	15.95
B-37	0.39812	7.55575	Yes	0.19446	14.64076	No	284	14.17
B-38	0.43407	7.5346	Yes	0.1941	15.13102	No	245	27.68
B-39	0.5837	7.68846	Yes	0.99023	16.08869	Yes	166	26.52
B-40	0.55904	7.93977	Yes	0.35622	14.72745	No	274	13.67
B-41	0.4805	7.97799	Yes	0.39157	14.49469	No	100	24.88
B-42	0.56964	7.95148	Yes	0.34132	15.9031	Yes	281	14.02
B-43	0.63898	7.93475	Yes	0.05476	15.40798	No	317	15.82
B-44	0.63813	7.85886	Yes	0.35535	15.69106	Yes	299	15.13
B-45	0.01451	7.7271	Yes	0.4113	15.51186	No	13	17.81
B-46	0.65953	7.29756	No	0.77371	15.33028	No	285	15.54
B-47	0.73136	7.55812	Yes	0.91155	14.8777	No	300	18.60
B-48	0.70544	7.35777	No	0.05707	15.44599	No	305	15.22
B-49	0.78022	7.25646	No	0.27936	15.31503	No	335	22.64
B-50	0.81366	7.14599	No	0.29563	15.32779	No	320	25.48
B-51	0.70398	6.99327	No	0.80813	16.07668	Yes	262	13.07
B-52	0.84097	6.91726	No	0.86368	14.75111	No	308	28.87
B-53	0.86675	6.83202	No	0.44803	14.65215	No	252	28.70
B-54	0.87565	6.72711	No	0.46809	14.98397	No	196	27.07
B-55	0.81275	6.65191	No	0.78144	15.38498	No	129	25.10
B-56	0.58349	6.60985	No	0.40651	14.68032	No	84	23.33
B-57	0.48579	6.63335	No	0.38862	15.26839	No	248	12.38
B-58	0.52343	6.65905	No	0.32554	14.58633	No	274	13.67
B-59	0.08023	6.62594	No	0.41736	14.88753	No	42	28.00
B-60	0.02312	7.26075	No	0.47181	15.71867	Yes	4	44.44
B-61	0.24163	7.29074	No	0.95866	15.9121	Yes	293	18.58
B-62	0.23794	6.80664	No	0.69733	14.66759	No	268	13.37

(*continued*)

Table 3.

Table 3.

Denomination	RMSE NS Bond excluded	Rest of bonds	NS best	RMSE SW Bond excluded	Rest of bonds	SW best	Arbitrage selection Exclusions	Exclusion days (%)
B-63	0.44258	7.18153	No	0.8583	15.18769	No	243	28.62
B-64	0.02495	7.4099	No	0.74197	15.06721	No	36	31.30
B-65	1.31352	7.30211	Yes	0.04455	15.86409	Yes	266	13.27
B-66	5.12752	8.44805	Yes	3.58675	15.36289	No	269	13.42
B-67	7.96396	9.43344	Yes	8.9904	14.89573	No	280	13.97
B-68	4.43874	9.7038	Yes	9.85891	14.79527	No	276	13.77
B-69	7.23281	10.49977	Yes	11.89666	14.99497	No	308	25.67
B-70	7.81231	10.92736	Yes	14.16842	15.89197	Yes	141	29.01
B-71	10.56599	10.72359	Yes	17.37316	15.85566	Yes	351	20.01
B-72	6.70971	8.14141	Yes	21.45581	15.73525	No	368	32.39

for coupons. In brief, the choice of the LLP as maturity does not seem an optimal criterion in the sample selection is to estimate the zero coupon curve.

Finally, Figure 1 presents the term structures of the interest rates for every model and sample using the mean value of estimated parameters.

The figure above shows that it is at the 30-years maturity when all models and samples seem to converge at the same long-term interest rate. However, in the prior maturities, there is clearly an important difference between models (NS and SW) and samples. Therefore, we consider that applying the term of 20 years as the LLP, as proposed by the EIOPA, together with the SW model, and not establishing a criterion for the selection of bonds to include when estimating the curve, brings with it considerable errors in the short, medium and long term (up to 30 years).

Sample	Nelson–Siegel	Smith–Wilson
Full sample	7.25735	15.69037
Best sample	7.16706	13.98115
Worst sample	8.88605	16.3787
Best-arbitrage	6.98761	13.4511
Worst-arbitrage	9.05699	17.02463

Table 4. Estimation errors for each interest rate model and difference samples

Parameters	Minimum	Q1	Q2	Q3	Maximum	Minimum bonds	Maximum bonds
All Samples							
$\beta 0$	−0.34909	0.02888	0.03313	0.03609	0.04393	37	48
$\beta 1$	−0.42009	−0.03951	−0.03611	−0.03202	0.01656		
$\beta 2$	−0.05207	−0.04505	−0.03917	−0.03344	0.03546		
τ	2.00335	2.39782	2.5863	2.75267	3.14446		
Worst Sample							
$\beta 0$	−0.39235	0.02495	0.02785	0.03119	0.05404	24	28
$\beta 1$	−0.62721	−0.03128	−0.02808	−0.02377	−0.01069		
$\beta 2$	−0.10552	−0.05254	−0.04626	−0.04117	3.99928		
τ	1.17326	1.5601	1.68842	1.83511	2.41211		
Best Sample							
$\beta 0$	−0.06997	0.03137	0.03455	0.03707	0.05212	11	20
$\beta 1$	−0.31097	−0.04618	−0.04262	−0.03922	0.01414		
$\beta 2$	−0.04578	−0.01645	−0.00104	0.02048	0.09369		
τ	3.7115	4.62317	5.8008	6.66616	7.59958		
Worst Arbitrage							
$\beta 0$	−0.21649	0.02737	0.03518	0.03764	0.0477	18	23
$\beta 1$	−0.04914	−0.04135	−0.03462	−0.03076	−0.02501		
$\beta 2$	−0.0548	−0.04937	−0.03705	−0.03177	0.00037		
τ	1.91939	2.31373	2.65896	2.79395	2.98571		
Best Arbitrage							
$\beta 0$	−0.02339	0.02931	0.03314	0.03653	0.0477	27	45
$\beta 1$	−0.01479	−0.03913	−0.0355	−0.03143	−0.02501		
$\beta 2$	−0.28494	−0.04433	−0.03793	−0.03273	0.00037		
τ	2.27627	2.41775	2.62347	2.7666	2.98571		

Table 5. Parameters of Nelson–Siegel model from difference samples

Table 6.
Parameters of Smith–Wilson model from difference samples

Parameters	Minimum	Q1	Q2	Q3	Maximum	Minimum bonds	Maximum bonds
All Samples							
α	0.06743	0.0831	0.1052	0.1177	0.12599	37	48
URF	0.01336	0.02316	0.02813	0.03167	0.04354		
Worst Sample							
α	0.05072	0.07024	0.09896	0.08731	0.0898	25	34
URF	0.01284	0.0216	0.02653	0.03077	0.04266		
Best Sample							
α	0.06821	0.08577	0.10548	0.12284	0.12927	12	19
URF	0.01357	0.02393	0.02887	0.03254	0.04366		
Worst Arbitrage							
α	0.07411	0.10672	0.11299	0.12038	0.13902	18	23
URF	0.01887	0.0316	0.03807	0.03391	0.05331		
Best Arbitrage							
α	0.06288	0.07305	0.06099	0.08996	0.11452	27	45
URF	0.01415	0.02533	0.03264	0.03522	0.04576		

Table 7.
Analysis of the relationship of bond maturity and coupon to the sample selection for the estimate of the interest rate curve

Parameters	Value	*t* Value	*t* Probability
Panel A. Nelson–Siegel best (logit)			
Constant	−1.7003	−2.87	0.005
Maturity	0.0866	2.67	0.01
Coupon	−4.8773	−0.278	0.782
Panel B. Smith–Wilson best (logit)			
Constant	−0.8682	−2.57	0. 0001
Maturity	0.0187	0.69	0.493
Coupon	−15.4132	−0.884	0.38
Panel C. Arbitrage exclusion days best (regression)			
Constant	128.3432	4.5693	0.0002
Maturity	4.7821	3.4617	0.0009
Coupon	8.6998	0.0937	0.9256

6. Conclusions

The EIOPA selects the SW model for its flexibility to fit the interest rate curve that should be applied by insurance companies. Moreover, the LLP is determined as the criterion to select the bonds that should be included in the sample estimate of the aforementioned curve. Specifically, this point is fixed in 20 years of maturity for the Euro curve. Finally, the EIOPA recommended that the curve parameters should be 0.1 for reversion speed to the long-term rate and 4.2% for the long-term rate.

We conducted a comparison study of the results of the SW and the NS models applied to daily data of bonds issued by the Government of Spain covering the period from January 1, 2014, to June 30, 2019. Several conclusions can be drawn from the findings of our study.

First, we found in terms of the parameter estimate of the model that the NS approach seems to be more flexible than the SW model, as it fits the three sections of the curve (short, medium and long term) by means of three parameters simplifying companies' risk management.

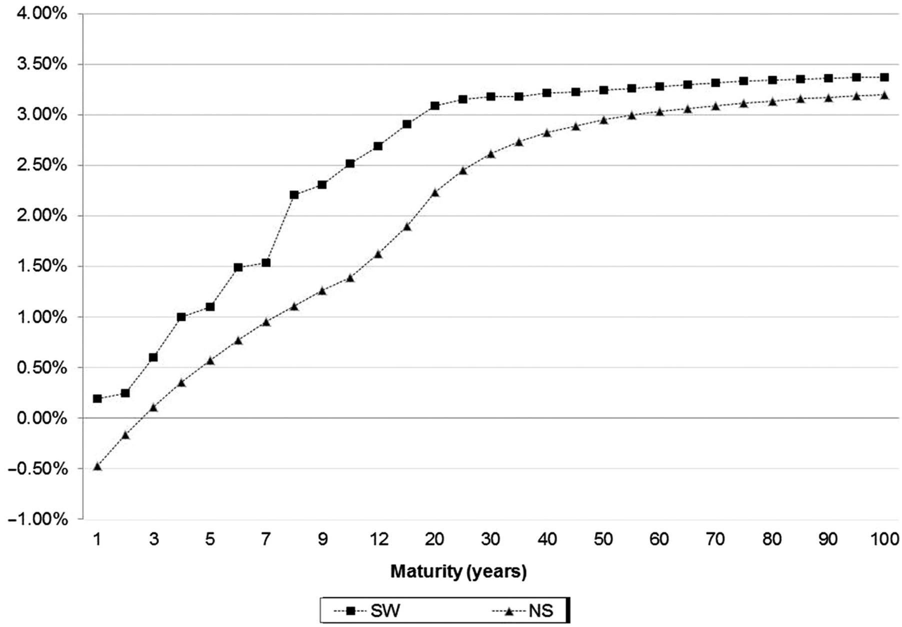

Figure 1.
Term structure of interest rate using median of estimated parameters

Second, we verified that the estimate error made by SW is always higher than that of the NS model, mainly in the interest rate curve's short term. The estimated parameters using the SW model are only similar to those recommended by the EIOPA for the values close to the maximum daily value, not for the average values. In addition, we found that the SW model is much more computing-intensive than the NS model, following González-Sánchez (2018).

Third, we observed that the sample selection used to estimate the interest rate curve must be independent of the model considered; otherwise, an endogeneity problem arises along with an increase in model risk. Likewise, we confirmed that the convergence between the different models and samples only occurs after 30 years. We analyzed the relationship between the coupon and the maturity of bonds excluded from the optimal samples. The results indicate that the SW model does not show a significant relationship, whereas the NS model and the selection by arbitrage show a positive relationship, i.e. the longer the bond maturity, the greater the possibility that the bond may be excluded from the optimal sample, although this relationship never converges at 20 years.

Our results are in line with Jørgensen (2018), so we observe how the SW model assumes an artificial increase in the interest rate curve, which would imply an undervaluation of financial liabilities on the balance sheets of insurance companies. Although our main results should be interpreted with caution, they do shed light on some issues related to estimating the interest rate curve in European insurance companies, which plays a crucial role in risk assessment and pricing. The empirical evidence shows that both the model and the parameters set by the EIOPA for the interest rate curve deviate from the optimal values, leading to an undervaluation of liabilities and going against the Solvency-II criteria, which in its article 75 clearly expresses the need to make valuations consistently with the market. In this regard, this paper could assist both the regulator and the insurance companies when choosing the optimal bonds in the curve estimate, as well as the curve model to be estimated.

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