METHODOLOGIES FOR THE TUNING OF PID CONTROLLERS IN THE FREQUENCY DOMAIN

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Abstract: This paper collects the authors experience about the tuning of PID controllers in the frequency domain and suggests solutions to several of the outlined problems. The paper presents: (1) a tuning methodology of non-interactive PID controllers by phase or gain margin, whose advantages becomes evident when a process model is available, and (2) a formulation of the combined tuning by phase and gain margin, in which priority is given to the phase margin. The graphic interpretation of the combined tuning shows great difficulties for the numeric methods and it can be used to establish a strategy on how to relax gain margin with the purpose of finding a solution.

Keywords: Frequency-domain, gain margin tuning, PID control, phase margin tuning, phase and gain margin tuning.

1. INTRODUCTION

The tuning of PID controllers in the frequency domain is a topic of great interest in the academic environment as well as in the industry field. Traditionally there has been a great difference between the academic and the industrial focus, because in the first case the tuning is solved under the hypothesis that the frequency response of the process is known, while in the second only a partial knowledge of such response is had. Nowadays this difference is smaller and smaller, because it is observed that in most of the cases a process model is used, and therefore the frequency response can be known. This type of tuning has given good results when applying it to processes of the refineries of the REPSOL group, through the tool SINTOLAB (Morilla, et al., 1996), because a total knowledge of the process model is had and the choice of the frequency design is made in an automatic or semiautomatic way.

This paper collects the experience of authors in the tuning of PID controllers in the frequency domain and it suggests solutions to several of the outlined problems. The possibility of combining two specifications (the phase margin and the gain margin) in the tuning is also looked at. This problem has been approached recently by Ho, et al. (1995), (1997) and (1998), Shafiei, and Shenton (1997), Fung, et al. (1998), Wang, and Shao (1999) but only for particular models or controllers. All the proposals are approached from the controller's point of view, without making any excessively restrictive hypotheses on the process model that limits the application of the proposed solutions.

In section 2 the general tuning method proposed by Aström, and Hägglund (1995) is presented, how this formulation can be used for tuning by phase margin or gain margin, and the tuning combined by phase and gain margin is approached. At the end of section 2, a critical revision of main problems arising in the tuning of the PID controllers in the frequency domain follows. In sections 3 and 4 specific solutions to some of these problems are proposed and tested with a non minimum phase process. Finally the paper is completed with the conclusions in section 5.

2. PID TUNING OF CONTROLLERS IN THE FREQUENCY DOMAIN

Aström and Hägglund (1995) have proposed a method for tuning PID controllers in the frequency domain which is a generalisation of the Ziegler-Nichols tuning rules, Figure 1. For PID non-interactive algorithms, described by the transfer function

$$G_C(s) = K_P\left(1 + \frac{1}{T_I\, s} + T_D\, s\right) \quad (1)$$

the determination of its parameters is summarised in the following expressions:

$$K_P = \frac{r_b \cos\left(\phi_b - \phi_a\right)}{r_a} \quad (2)$$

$$T_I = \frac{1}{2\,\alpha\,\omega_c}\left(\mathrm{tg}\left(\phi_b - \phi_a\right) + \sqrt{4\,\alpha + \mathrm{tg}^2\left(\phi_b - \phi_a\right)}\right) \quad (3)$$

$$T_D = \alpha \, T_I \quad (4)$$

Where: ω_c is the design frequency; r_a and ϕ_a are the module and the phase of point A; r_b and ϕ_b are the module and the phase of point B. And α is the ratio between the derivative T_D and the integral T_I time constants. The value of α should be specified to obtain a solution of among all the possible ones.

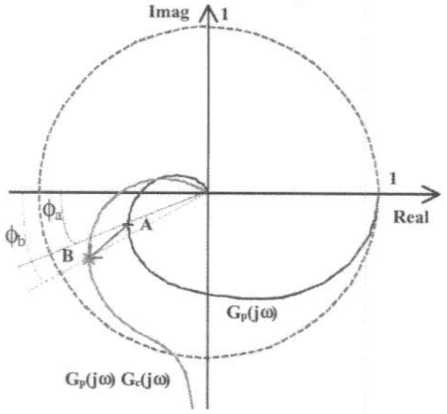


Fig.1. Nyquist diagram where the PID controller have been tuned to move the point A to B.

2.1 Tuning by phase or gain margin.

The Aström and Hägglund´s method (1995) includes as a particular case the tuning by phase margin. When a phase margin ϕ_m is specified, the following conditions are imposed to point B: $r_b=1$, $\phi_b=\phi_m$. This kind of tuning has already been considered by Aström, and Hägglund (1984), but not starting at an arbitrary point A but from the ultimate point of the Nyquist diagram that is on the real axis ($r_a=1/K_u$, $\phi_a=0$). Aström and Hägglund´s (1995) method can also be used to tuning the PID controller by a gain margin A_m. The following conditions are imposed to point B: $r_b=1/A_m$, $\phi_b=0°$. It can be shown that the gain margin tuning proposed by Aström, and Hägglund (1984) is a particular case on the general procedure, starting at the ultimate point.

2.2 Combined tuning by phase and gain margin.

While in expressions (2), (3) and (4) no condition is imposed to point B, and therefore to the behaviour of the closed loop system, in the tuning by phase or gain margin a condition of relative stability is imposed. But a specification $\phi_m>0$ does not guarantee that the resulting ϕ_m is positive and that therefore the system is stable. In the same way, a specification $A_m>1$ does not guarantee that the resulting system is stable. The only way of guaranteeing the stability is to combine both specifications ($\phi_m>0$ and $A_m>1$).

The combined tuning by of phase (ϕ_m) and gain (A_m) margin can be interpreted as it is shown in Figure 2, as the movement of two points of the Nyquist diagram from positions A and D to positions B ($r_b=1$, $\phi_b=\phi_m$) and E ($r_e=1/A_m$, $\phi_e=0$). If both specifications have been reached with the same control parameters, it is because the points A and D verify the following two expressions:

$$\frac{\cos(\phi_m-\phi_a)}{r_a} = \frac{\cos(-\phi_d)}{A_m \, r_d} \quad (5)$$

$$\frac{1}{\omega_a}\left(tg(\phi_m-\phi_a)+\sqrt{4\alpha+tg^2(\phi_m-\phi_a)}\right) = \frac{1}{\omega_d}\left(tg(-\phi_d)+\sqrt{4\alpha+tg^2(-\phi_d)}\right) \quad (6)$$

Being: ω_a, r_a and ϕ_a the frequency, the magnitude and the phase of point A, ω_d, r_d and ϕ_d the frequency, the magnitude and the phase of point D. Therefore the tuning by phase and gain margin will be possible if a couple of frequencies ω_a and ω_d exist in such a way that the two previous expressions are verified. The control parameters can be determined later on, making use of the tuning expressions for phase margin with $\omega_c=\omega_a$ as design frequency.

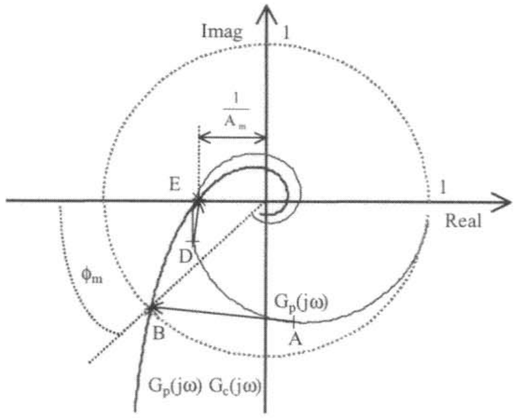


Fig. 2. Example of a PID controller tuning by phase and gain margin.

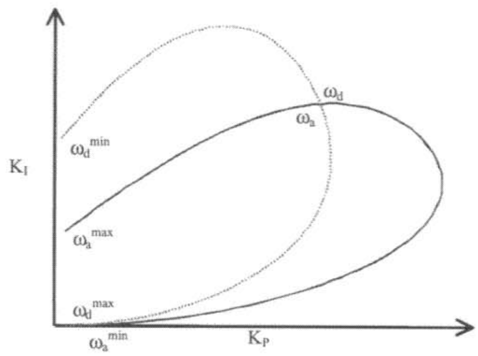


Fig. 3. Example of PID tuning by phase margin and gain margin in the parametric control plane.

The tuning with specifications of phase and gain margin can also be interpreted in the K_P-K_I plane, in a similar way to that proposed by Shafiei, and Shenton (1997). The continuous curve in Figure 3 represents the set of control parameters with which it is possible to obtain a specific phase margin and the dashed curve represents the set of control parameters with which it is possible to obtain a specific gain margin. The intersection of the two curves represents the set of control parameters with which it is possible to get both specifications at the same time. The intersection point will have associated the frequency ω_a in the curve of the phase margin and the frequency ω_d in the curve of the gain margin.

2.3 *Interesting problems.*

The tuning by phase or gain margin is not completely solved with a simple calculation of the control parameters. Before arriving to this situation it has been necessary to solve other problems as: the election of points A and B, the type of controller and the ratio between T_D and T_I. In section 3 specific solutions to some of these problems are proposed when a linear process model is available.

The combined tuning by phase and gain margin, that only can be approached when a process model is available, neither is reduced to solve a system of two non-linear equations. Before arriving to this situation it has been necessary to solve other problems as: the election of points B and E in Figure 3, the type of controller and the ratio between T_D and T_I. But on the way several questions immediately arise: under which conditions in the specifications is there a solution to the equations?, is the solution unique?, how to find it?, if a solution with some certain specifications is not found which specification ϕ_m or A_m should relax first? The section 4 try to answer these questions, sometimes in a general way and others for specific cases.

3. THE SPACE OF SOLUTIONS FOR TUNING BY PHASE OR GAIN MARGIN

If a process model is known, the point A can be any point of the Nyquist diagram. Then will be the controller (PI, PD, PID) that imposes an angular condition to points B in order to make the tuning possible. The rotation, of value $\phi_c=\phi_b-\phi_a$, should be bounded between the minimum and the maximum phase that the controller can contributes, see Figure 4. Otherwise when the point B has been selected, the point A cannot be selected arbitrarily. This problem can be divided in two: a) which is the range of the possible frequencies design? b) what value inside that range should be chosen as frequency design? These will be analyzed in the next two sections.

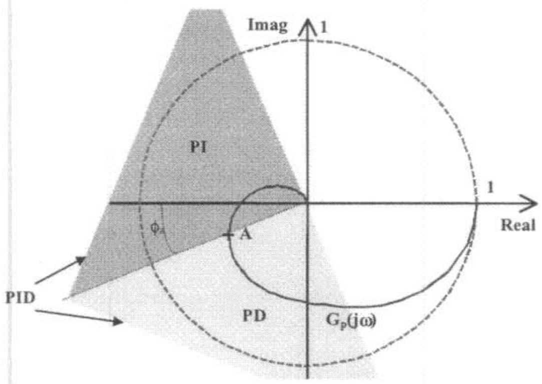


Fig. 4. Domains (dark zones) of the possible points B when point A has been arbitrarily chosen.

3.1 *Determination of the range for the design frequency ω_c.*

The possible points A are only those points of the process frequency response that fulfills the angular condition. They define a range of frequencies for which the calculation of the control parameters is possible. In other words, ω_c should fulfill the condition

$$\phi_c^{min} \le (\phi_b - 180^o - \arg(G_p(j\omega_c))) \le \phi_c^{max} \qquad (7)$$

where ϕ_c^{min} and ϕ_c^{max} depend of the controller. In Figure 5 an example is shown. In this example it has been considered that point B corresponds to a specification of phase margin equal to 45º, or any other point of the bisector of the third quadrant, and the controller is PID type.

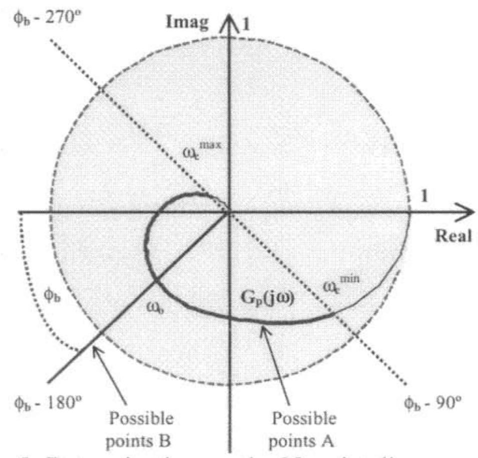


Fig. 5. Determination on the Nyquist diagram of the range for the design frequency.

3.2 *ω_c determination.*

As the bandwidth of the system in closed loop is directly related with the design frequency, must the following question be asked: why not choosing the maximum possible frequency? One of the reasons might be that the bigger the design frequency is the riskier it is to destabilize the system. Another reason is that in the case of a PI controller, the value of ω_c^{max} would lead us to an infinite value of the integral time, that is to say, to a controller with just a proportional action. In the case of a PD controller, it would lead us to a controller with just one derivative action. And finally in the case of a PID controller, we would have a controller with the maximum derivative action.

In this section it is intended to use an auxiliary criteria jointly with the specification (phase or gain margin) for the automatic election of the design frequency. This approach is not the only possible one, but it is followed by other authors (Aström, and Hägglund, 1995; Shafiei, and Shenton, 1997). For PI and PID controllers it is intended to choose the design frequency that allows to get a maximum integral gain ($K_I = K_P/T_I$). Figure 6 shows an example of some K_I graphs versus ω_c obtained when a specification of phase margin equal to 45° (r_b=1, ϕ_b=45) and the controller is PID with different values for α, included the PI case (α=0). It is observed a maximum in all cases, which can be valid to choose the design frequency.

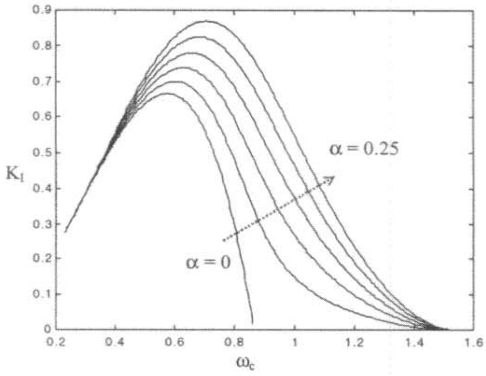


Fig. 6. Integral gain as function of the design frequency.

3.3 Example: Application to a non minimum phase system.

The previous proposals have been successfully applied to a big number of processes considered in the bibliography on PID controllers. As an example of this, the application to a non minimum phase system is presented. It has been used by Ho, et al. (1995) to prove their design methods and tuning formulas. The transfer function of this model is

$$\frac{1 - \mu s}{(s + 1)^3} \qquad (10)$$

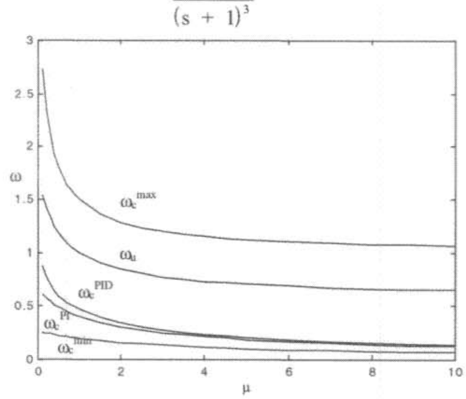


Fig. 7. Range and possible values for the design frequencies.

Figure 7 shows the relative situation of several design frequencies regarding the range $]\omega_c^{min}\ \omega_c^{max}[$ for a PID controller for phase margin equal to 45°, when $1 \geq \mu \geq 0.1$. The lower curve corresponds to the ω_c^{PI} frequencies, obtained for a PI controller with the objective of maximising K_I. The immediately superior one corresponds to the ω_c^{PID} frequencies, obtained for a PID controller when imposing T_D/T_I=0.1. The ω_u curve corresponds to the ultimate frequencies that do not depend on the type of controller which is used. It is observed that starting from μ=3, there is even very little difference between the election of PI or PID controllers.

Table 1 Parameters and characteristics in the situations of Figure 8.

μ	Controller	K_P	T_I	T_D	ω_{cp}	A_m	ω_{cg}	IE
0.2	PI	1.17	2.14	0	0.57	2.60	1.12	1.82
0.2	PID	1.92	2.81	0.28	0.77	3.34	1.63	1.47
2	PI	0.59	2.50	0	0.31	1.46	0.63	4.22
2	PID	0.71	2.79	0.28	0.34	1.46	0.73	3.94

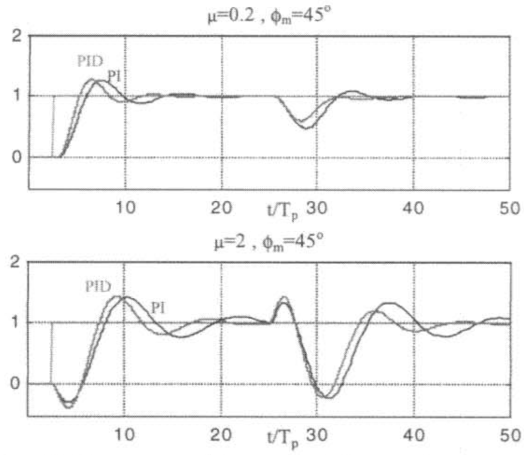


Fig. 8. Responses of the non minimum phase process with PI and PID tuning by phase margin.

Figure 8 shows the time responses for two process (μ=0.2 and 2) and two controllers (PI, PID) with specification of phase margin equal to 45°. The control parameters used are shown in Table 1, together with the characteristics of the frequency response. It is observed that the PI and PID responses differ from each other, as it could be expected from Figure 7. The results are goods even when the difficulty of control increases (μ=2). In Figure 8 the results with the tuned control parameters starting from the ultimate frequency are not included because they give place to greater integral time constants than the other tunings, with the corresponding increase of the error integral for load changes. And because, starting from μ=2, situations of unstability (negative gain margin) take place.

4. THE SPACE OF SOLUTIONS FOR TUNING BY PHASE AND GAIN MARGIN.

If a numeric or graphic method is going to be used in the resolution of equations (5) and (6), it is convenient to have bounded the solution space, that is to say, the space of possible couples (ω_a,ω_d) that can be a solution. Is this possible? The answer is affirmative. As the tuning proposed is a combination of tuning by phase and gain margin, the possible couples of frequencies ω_a and ω_d that verify (5) and (6) belong respectively to the possible ranges $]\omega_a^{min}\ \omega_a^{max}[$ for the specified phase margin and $]\omega_d^{min}\ \omega_d^{max}[$ for phase margin equal to zero. This delimitation of the problem in the frequency domain presents the following advantages: 1st) As the phase is generally monotonous decreasing with the frequency, it will be fulfilled that $\omega_a^{min}<\omega_d^{min}<\omega_a^{max}<\omega_d^{max}$. 2nd) A change in the specified phase margin will modify the range for ω_a, but a change in the gain margin will not affect to the range ω_d. 3rd) A change in the value of α, except for the change of PI to PID control does not modify the ranges for ω_a and ω_d.

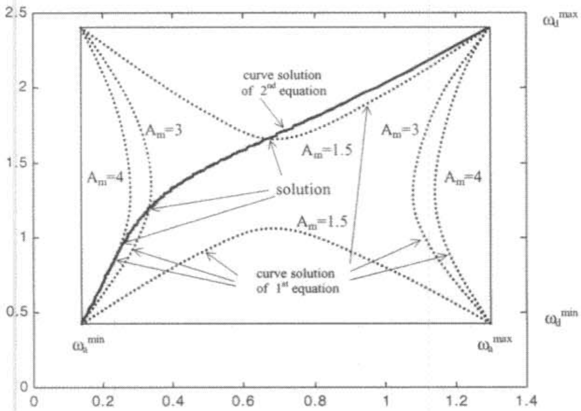


Fig. 9. Analysis of tuning by ϕ_m=60° and several gain margins for the non-minimum phase process (μ=1) with PID control and α =0.25.

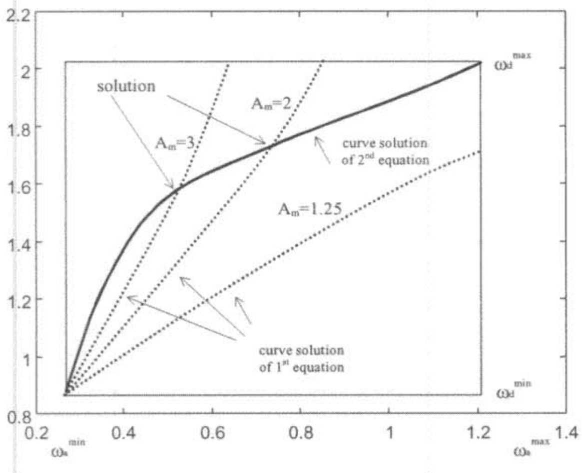


Fig. 10. Analysis of tuning by ϕ_m=60° and several gain margins for the process described by $G(s)=e^{-s}/(s+1)$ with PI control.

As an example, Figure 9 shows graphically the solutions of equations (5) and (6), in several situations, characterised by the same phase margin and different gain margin. The couples (ω_a,ω_d) that are solutions of the second equation (6) are located in a single curve of the plane. The curve defines two regions in the space of solutions. The couple of curves which are a solution of the first equation (5) define three regions in the space of solutions: either horizontal regions as is the case for A_m=3, or vertical regions as is the case for A_m=1.5. On the other hand, if the controller is PI or PD, all the solutions to the equation (5) will belong to a curve. This curve will be located above the bisector of the space of interest when the maximum of the term on the left is smaller than that on the right and vice versa. That is what has caused the different curves solution of the first equation in Figure 10.

4.1 Solution to the system of equations.

A detailed graphic analysis of the equations (5) and (6) allows to affirm that:

- The equations may have a clear solution. For example in Figure 9, for A_m=3 or A_m=1.5, the curves solution intersect in a specific point.
- The equations may have a solution in a range of frequencies. For example in Figure 9, for A_m=4, the curves solution are very close in a wide area.
- There are always solutions near the left lower extreme and sometimes near the right upper extreme of the space of solutions. But these solutions should not be used, since they correspond to not very usual situations, where a proportional, integral or derivative action dominates over the other ones. If a numeric method is used a great probability of falling in these local minima exists.
- The equations might not have a solution, except for some of the extremes. For example in Figure 10, for A_m=1.25. In these cases the numeric method would go to the relative minimum, but the graphic method allows to state a strategy on which specification should be relaxed to find a solution.

Although the analysed cases do not contemplate the whole complete casuistry, it can be concluded that the resolution of (5) and (6) is not trivial, at least in a numeric way, but needs a graphic analysis. This analysis should be carried out in the space of frequencies that results from combining the ranges for phase and gain margin tuning.

4.2 Example: Application to the non minimum phase system.

The non minimum phase process with μ=1 has been also used by Ho, et al. (1995) to check their tuning formulas by phase and gain margin. They need a second order system with dead time, so they are forced to use one intermediate model with two time constants. However, this intermediate model are not necessary to apply the methodology described in this paper. The three values (4.0, 3.0 and 1.5) of A_m in Figure 9 have been chosen after analysing Figure 11, that shows the possible gain margins in function of the possible design frequencies for the tuning by phase margin equal to 60°.

Figure 12 shows the time responses for the same process (μ=1) and three situations in the controller (PID1, PID2, PID3) tuned with the same specification of phase margin equal to 60° but different specifications of gain margin: 4.0, 3.0 and 1.5 respectively. The control parameters used together with the characteristics of the frequency response, are shown in Table 2.

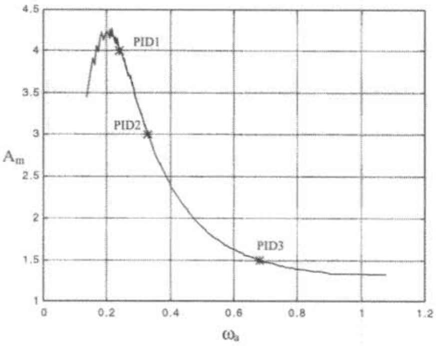
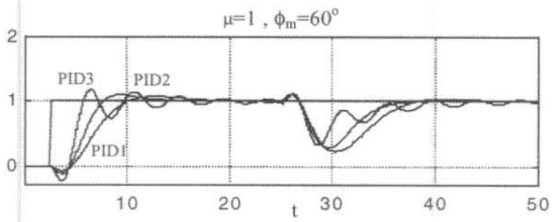


Fig. 11. Possible gain margins for the non minimum phase process (μ=1) with PID control and T_D/T_I=0.25 for ϕ_m=60°.

Table 2 Parameters and characteristics in the situations of Figure 12 and when applying the formulas of Ho, et al. (1995).

ϕ_m=60°	K_P	T_I	T_D	ϕ_m	ω_a	A_m	ω_d	IE
PID1	0.43	1.77	0.44	60.0	0.24	4.0	0.88	4.12
PID2	0.76	2.39	0.60	60.0	0.33	3.0	1.19	3.14
PID3	1.40	3.96	1.00	60.0	0.68	1.5	1.68	2.83
PIDH1	0.50	1.96	0.49	60.9	0.25	3.92	0.98	3.92
PIDH2	0.66	2.00	0.50	53.5	0.33	3.03	1.00	3.03
PIDH3	0.82	-7.81	1.13					



Fig, 12. Responses for the non minimum phase process with different PID controllers.

The situations PIDH1, PIDH2 and PIDH3 when the parameters are tuned by the formulas of Ho et al. (1995) are also included in Table 2. The PIDH3 situation give place to a negative value of the integral time constant that can be interpreted as if ϕ_m=60° and A_m=1.5 cannot be obtained; however the general method does give a solution in this situation. It is observed that the specifications are obtained with the PID1, PID2 and PID3 tunings, since the general method is an exact method, while with the formulas of Ho, et al. (1995) some characteristics close to the specifications are obtained, because of the formulas are approximate and in this case an intermediate model is also needed.

5. CONCLUSIONS

This paper shows that the PID controllers tuning in the frequency domain is not an easy task. A methodology for the tuning non-interactive PID controllers by phase or gain margin has been presented. The advantages of using this methodology become evident when a process model is available. And since restrictive hypothesis on the process model are never made, the result is that the methodology is applicable to a great variety of processes: with or without a dead time, with a dominant time constant, with a dominant dead time, with or without an integrator, with minimum or non-minimum phase responses, with overdamped or underdamped responses, etc...

A formulation of the combined tuning by phase and gain margin, in which priority is given to the phase margin, is also presented. The graphic interpretation shows great difficulties for the numeric methods and it can be used to establish a strategy on how to relax gain margin with the purpose of finding a solution.

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