IDENTIFICATION FOR ROBUST CONTROL OF A FAST FERRY

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Abstract: The interval transfer functions from wave height to pitch and heave movement described in this paper are interpreted as a family of transfer functions whose coefficients are bounded by some know intervals and centred at nominal values. The nominal model is obtained by a non-linear least square algorithm of identification applied in the frequency domain. Once the nominal model was obtained, then the tightest intervals around each coefficient of the nominal transfer functions was created while satisfying the membership and frequency response requirements. Different model validation tests was made (Bode plots and simulations). These tests show that the uncertainty model obtained is a valid interval model and it can be used for robust control design.

Keywords: Identification algorithms, Optimization problem, Robust performance.

1. INTRODUCTION

The main problem for the development of high speed ship is concerned with the passenger's comfort and the safety of the vehicles. The vertical acceleration associated with roll, pitch and heave motion is the cause of motion sickness. The roll control is the most attractive candidate for control since increasing roll damping can be obtained more easily. However, shipbuilders are also interested in increasing pitch and heave damping. In order to solve the problem antipiching devices and pitch control methods must be considered. Previously, models for the vertical ship dynamic must be developed for the design, evaluation and verification of the results.

The number of published investigations about ship modelling is inmense. For example, nonlinear models in 6 degrees of freedom are shown in Fossen (1994) and Lewis (1989). These models are theoretical and they are obtained from the equations of a rigid solid partially inmersed in water.

Obtaining a very accurate mathematical model of a system is usually impossible and very costly. It also often increases the complexity of the control algorithm. A trend in the area of system identification is to try to model the system uncertainties (Bhattacharyya et al., 1995) to fit the available analysis and design tools of robust control.

The interval functions described in this paper are interpreted as a family of transfer functions from wave height to pitch and heave movement whose coefficients are bounded by some know intervals and centred at nominal values. The nominal model (Aranda et al., 1999b; Aranda et al., 2000) is obtained by a non-linear least square algorithm applied in the frecuency domain. Once the nominal model is obtained, then the tightest intervals around each coefficient of the nominal transfer functions are created while satisfying the membership and frequency response requirements.

2. IDENTIFICATION METHODOLOGY

The method describes in this paper follows the steps of classical identification diagram (Ljung, 1989; Schoukens and Pintelon, 1991; Södertröm and Stoica, 1989). A model test was carried out in the towing tank of CEHIPAR (Madrid, Spain). The model was free to move in heave direction and pitch angle. The wave surface elevation was measured at 68.75 m. forward from model bow. Different regular and irregular waves and ship speed were tested. A set of simulated data (Aranda et al., 1999a) has been generated by the program PRECAL (which uses a geometrical model of the ship to predict her dynamic behaviour), reproducing the same conditions of the experiments with regular waves.

Two transfer functions are identified (see Figure 1):

- $G_P(s)$: transfer function from wave height (m) to pitch movement (º).
- $G_H(s)$: transfer function from wave height (m) to heave movement (m).

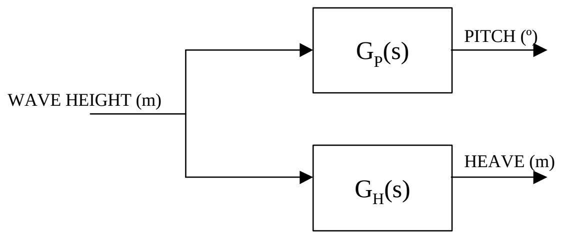

Fig. 1. Blocks diagram of the identified system

The identification is made in the frecuency domain and uses the simulated data of magnitude and phase obtained by the program PRECAL in the encounter frequency ω_{ei} (i=1,2,...,25) for the transfer functions $G_P(j\omega_{ei})$ and $G_H(j\omega_{ei})$.

$$G_P(jw_{ei}) = \mathrm{Re}(G_P(jw_{ei})) + j\,\mathrm{Im}(G_P(jw_{ei}))$$
$$G_H(jw_{ei}) = \mathrm{Re}(G_H(jw_{ei})) + j\,\mathrm{Im}(G_H(jw_{ei}))$$

In general, the estimated transfer functions $\hat{G}_P(s)$ and $\hat{G}_H(s)$ can be written in the following form:

$$\hat{G}(s) = \frac{x_{n+m+1}s^m + x_{n+m}s^{m-1} + \ldots.. + x_{n+1}}{s^n + x_n s^{n-1} + \ldots. + x_1} \qquad (1)$$

where m is the number of zeros and n is the total number of poles. The parameter vector is:

$$\vec{P} = (x_1, x_2, \ldots, x_n, x_{n+1}, \ldots, x_{n+m+1}) \qquad (2)$$

The estimation of the parameter vector $\vec{P}$ is made by a non-linear least squares procedure that uses the following cost function (Schoukens and Pintelon, 1989):

$$K(\vec{P}) = \sum_{k=1}^{N} |(\mathrm{Re}(G(jw_{ek})) - (\mathrm{Re}(\hat{G}(jw_{ek})) + \ldots + j(\mathrm{Im}(G(jw_{ek})) - (\mathrm{Im}(\hat{G}(jw_{ek}))|^2 \qquad (3)$$

A number of considerations need to be made based in a priori knowledge of the ship dynamics. So, there are three constraints in the identification process of the models:

- The models must be stables.
- The gain of $G_P(s)$ must tend to zero in low encounter frequencies.
- The gain of $G_H(s)$ must tend to one in low encounter frequencies

The solution to a non-linear least squares problem with constrains is described for example in Söderström and Stoica (1989), and can be programed using MATLAB.

3. INTERVAL MODELLING

Bhattacharyya et al. (1995) describes a method to obtain the family of linear time invariants systems $\vec{G}(s)$ by letting the transfer function coefficients lie in intervals around those of the nominal G(s). This method is adapted to our problem. Let

$$y(jw_{ei}) = D(jw_{ei})u(jw_{ei}) \quad i = 1, 2, \ldots, N \qquad (4)$$

where $\omega_{e1}, \omega_{e2}, \ldots, \omega_{eN}$ are the test encounter frequencies and the complex number $u(j\omega_{ei})$ and $y(j\omega_{ei})$ denote in phasor notation the input-output pair at the frequency ω_{ei} generated from an identification experiment. Suppose that $G^I(s)$ is the transfer function of a linear time-invariant system which is such that $G^I(j\omega_e)$ is closest to $D(j\omega_e)$ in some norm sense. In general it is not possible to find a single rational function $G^I(s)$ for which $G^I(j\omega_{ei})=D(j\omega_{ei})$ and the more realistic identification problem is to fact identify an entire family $\vec{G}(s)$ of transfer functions which is capable of validating the data in the sense that for each point $D(j\omega_{ei})$ there exists some transfer function $G_i \in \vec{G}(s)$ with the property that $G^I(j\omega_{ei})= D(j\omega_{ei})$.

Let the nominal transfer function $G^I(s)$, which has been identified by a non-linear least squares procedure explained in the previus section, and the transfer function G(s) with the form:

$$G(s) = \frac{\hat{x}_{n+m+1}s^m + \hat{x}_{n+m}s^{m-1} + \ldots.. + \hat{x}_{n+1}}{s^n + \hat{x}_n s^{n-1} + \ldots.. + \hat{x}_1} \qquad (5)$$

The family of linear time-invariant systems $\vec{G}(s)$ is defined by :

$$\vec{G}(s) = \{G(s) : \hat{x}_i \in [x_i - w_{x_i} \cdot e^-_{x_i}, x_i + w_{x_i} \cdot e^+_{x_i}]\ \forall i\} \qquad (6)$$

where w_{xi} are to be regarded as *weigths* chosen apriori whereas the ε's are to be regarded as *dilation parameters* to be determinated by the identification algorithm and the data $D(j\omega_{ei})$.

3.1 Weight selection

Suppose the test data consists of N data points obtained at corresponding frequencies,

$$D(jw_e) = \{D(jw_{ei}) = a_i + jb_i \ , i = 1,2,...N\} \quad (7)$$

the l^{th} model is defined as:

$$G_l(jw_e) = \begin{cases} D(jw_{ei}) & i = l \\ G^I(jw_{ei}) & i = 1,2,....., l-1, l+1,...., N \end{cases} \quad (8)$$

The model $G_l(j\omega_{ei})$ is identical to the nominal identified model $G^I(j\omega_{ei})$ with the l^{th} data point replaced by the l^{th} component of the test data $D(j\omega)$. Now the l^{th} identified model $G_l^I(s)$ is constructed, which is identified from the l^{th} data set $G_l(j\omega)$. Let

$$G_l^I(s) = \frac{x^l_{n+m+1}s^m + ...+ x_{n+1}^l}{s^n + x_n^l s^{n-1} ......+ x_1^l} \quad (9)$$

The models $G_l^I(s)$ must be identified with the same method used to identify the nominal model $G^I(j\omega)$. The weigth vector $\vec{w}$ is :

$$\vec{w} = \left[\frac{1}{N}\sum_{l=1}^{N}\left|x_1 - x_1^l\right|,......,\frac{1}{N}\sum_{l=1}^{N}\left|x_{n+m+1} - x^l_{n+m+1}\right| \right] \quad (10)$$
$$\vec{w} = [w_{x_1},......, w_{x_n}, w_{x_{n+1}},......, w_{x_{n+m+1}}]$$

The weigth selection is an important stage because an inappropriate selection may results in an unnecessarily large family.

3.2 Computation of the intervals of the transfer function coefficients.

Replacing $s=j\omega_{ei}$ in (5):

$$G(jw_{ei}) = \frac{\left(\hat{x}_{n+1} - w_{ei}^2\hat{x}_{n+3} + ...\right) + j\cdot\left(w_{ei}\hat{x}_{n+2} - w_{ei}^3\hat{x}_{n+4} + ...\right)}{\left(\hat{x}_1 - w_{ei}^2\hat{x}_3 + ...\right) + j\cdot\left(w_{ei}\hat{x}_2 - w_{ei}^3\hat{x}_4 + ...\right)} \quad (11)$$

if $G(j\omega_{ei})$ is made equal to the data set $D(j\omega_e)$ for a particular encounter frequency ω_{ei}, then:

$$D(jw_{ei}) = a_i + jb_i = \frac{n1 + j\cdot n2}{d1 + j\cdot d2} \quad (12)$$

Operating, the next pair of equations are obtained:

$$F_1\left(a_i, b_i, x_1^i,..., x^i_{n+m+1}\right) = (a_i d1 - b_i d2) - n1 = 0$$
$$F_2\left(a_i, b_i, x_1^i,..., x^i_{n+m+1}\right) = (b_i d1 + a_i d2) - n2 = 0 \quad (13)$$

$\hat{x}_i$ for all i is defined by:

$$\hat{x}_i = x_i + w_{x_i} e^{\,l}_{x_i} \quad \begin{cases} i = 1,....,n+m+1 \\ l = 1,......,N \end{cases} \quad (14)$$

Rewrite (14) in terms of a matrix equations:

$$A\cdot\vec{x} + A\cdot W\cdot\vec{e}^{\,l}_x = -E$$
$$A\cdot W\cdot\vec{e}^{\,l}_x = -B - E \quad (15)$$

where:

$$A = \begin{bmatrix} a_i & -b_i w_{ei} & a_i w_{ei}^2 & -b_i w_{ei}^3 & . & -1 & 0 & w_{ei}^2 & 0 & w_{ei}^4 & . & . \\ b_i & a_i w_{ei} & -b_i w_{ei}^2 & a_i w_{ei}^3 & . & 0 & w_{ei} & 0 & w_{ei}^3 & 0 & . & . \end{bmatrix}$$
$$E = \begin{bmatrix} k_1 w_{ei}^n \\ k_2 w_{ei}^n \end{bmatrix}$$
$$k_1 = \begin{cases} a_i \ \ si\ n = 0,4,8,.... \\ -b_i \ \ si\ n = 1,5,9,.... \\ -a_i \ \ si\ n = 2,6,10,.... \\ b_i \ \ si\ n = 3,7,11,.... \end{cases} \quad k_2 = \begin{cases} b_i \ \ si\ n = 0,4,8,.... \\ a_i \ \ si\ n = 1,5,9,.... \\ -b_i \ \ si\ n = 2,6,10,.... \\ -a_i \ \ si\ n = 3,7,11,.... \end{cases} \quad (16)$$
$$W = \begin{bmatrix} w_{x_1} & & & 0 \\ & . & & \\ & & . & \\ 0 & & & w_{x_{n+m+1}} \end{bmatrix}$$
$$\vec{e}^{\,l}_x = \left[e^{\,l}_{x_1} \ \ . \ \ . \ \ . \ \ . \ \ . \ \ . \ \ e^{\,l}_{x_{n+m+1}}\right]^T$$
$$\vec{x} = [x_1,................, x_{n+m+1}]^T$$
$$B = A\cdot\vec{x}$$

$\vec{e}^{\,l}_x$ is the vector of the dilation parameters obtained for the encounter frecuency ω_{el}. Here it is assumed without loss of generality that $A(\omega_{ei},\alpha_i,\beta_i)$ has full rank. Then the minimun norm solution $\vec{e}^{\,l}_x$ can be computed as:

$$\vec{e}^{\,l}_x = -W^{-1}\left(A^T A\right)^{-1} A^T\left(B + E\right) \quad (17)$$

After finding $\vec{e}^{\,l}_x$ for all l=1,...,N, the dilation parameters of the intervals of the transfer function coefficients are determined as follows:

$$e_{x_k}^{\ -} = \min_l\left\{0, e^{\,l}_{x_i}\right\} \quad e_{x_k}^{\ +} = \max_l\left\{0, e^{\,l}_{x_i}\right\} \quad (18)$$

4. RESULTS

In Table 1 and Table 2 different model structures (where m is the number of zeros, n is the total number of poles and nps is the number of simple poles) are showed for heave and pitch movement, at several ship speed. The cost function and mean square error can be compared when the model structure is reduced.

Table 1: Model structures for heave movement

Ship speed (knots)	Model Structure (m,n,nps)	Value of the cost function	Mean square error (m^2)
20	(4,6,2)	0.0383	0.0143
20	(3,5,1)	0.0692	0.0141
20	(2,3,1)	0.0696	0.0138
30	(4,6,2)	0.0385	0.0111
30	(3,5,1)	0.1012	0.0115
30	(2,3,1)	0.2381	0.0170
40	(4,6,2)	0.0471	0.0112
40	(3,5,1)	0.1045	0.0113
40	(2,3,1)	0.4510	0.0125

Table 2: Model structures for pitch movement

Ship speed (knots)	Model Structure (m,n,nps)	Value of the cost function	Mean square error $((°)^2)$
20	(4,6,2)	0.1213	0.1056
20	(3,5,1)	0.1228	0.1052
30	(4,6,2)	0.0938	0.0995
30	(3,5,1)	0.0946	0.0998
40	(4,6,2)	0.0942	0.1214
40	(3,5,1)	0.0989	0.1226

The model interval was obtained for each of model structures show in Table 1 and Table 2. For example, the transfer functions of model structure (4,6,2) for heave movement and pitch movement at 40 knots are:

$$G_H(s) = \frac{3.219s^4 - 0.9423s^3 + 26.03s^2 - 6.78s + 80.35}{s^6 + 16.43s^5 + 42.62s^4 + 106.6s^3 + 142.9s^2 + 142.6s + 80.35}$$

$$G_P(s) = \frac{0.5381s^4 - 6.051s^3 + 13.21s^2 - 52.28s}{s^6 + 9.855s^5 + 28.74s^4 + 63.99s^3 + 91.84s^2 + 83.73s + 50.08}$$

In Table 3 and Table 4 the model interval of $G_H(s)$ and $G_P(s)$ are showed.

Table 3: Model interval of $G_H(s)$

x	Lower Interval	Nominal value	Upper Interval
x_1	79.95	80.35	83.50
x_2	139.79	142.61	143.09
x_3	139.63	142.94	144.56
x_4	106.31	106.59	109.02
x_5	35.88	42.62	43.09
x_6	12.98	16.43	16.52
x_8	-6.81	-6.78	-6.28
x_9	25.99	26.02	26.35
x_{10}	-5.14	-0.92	-0.81
x_{11}	-0.14	3.21	3.28

Table 4: Model interval of $G_P(s)$

x	Lower Interval	Nominal value	Upper Interval
x_1	49.71	50.08	50.87
x_2	80.70	83.73	84.31
x_3	91.03	91.84	92.42
x_4	63.45	63.99	66.07
x_5	28.31	28.73	28.95
x_6	6.19	9.85	9.95
x_8	-53.07	-52.57	-52.48
x_9	12.55	13.21	13.47
x_{10}	-6.79	-6.05	-4.59
x_{11}	0.25	0.53	2.74

In Figure 2 Bode plot of $G_H(s)$ and data obtained by PRECAL are showed.

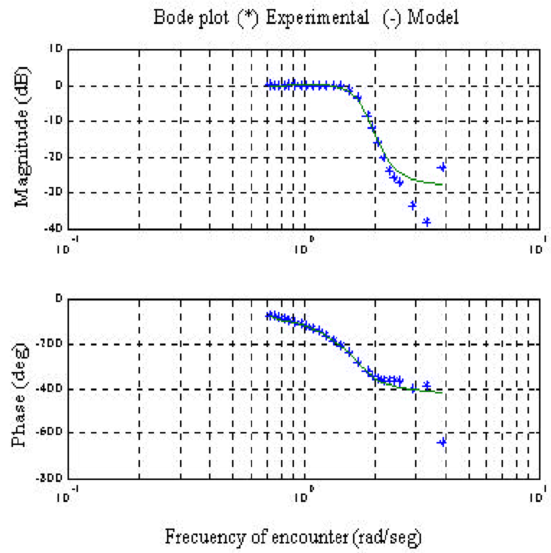

Fig. 2. Bode plot of $G_H(s)$ and data of PRECAL program.

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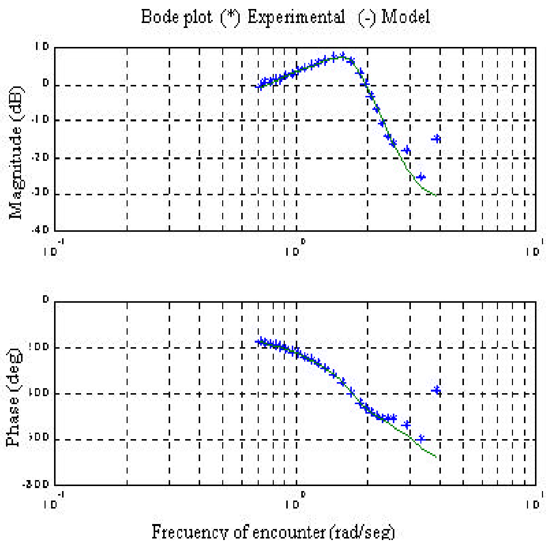

Fig. 3. Bode plot of $G_P(s)$ and data of PRECAL program.

Figure 4 shows the output of $G_H(s)$ and the measured heave in the CEHIPAR when the input was irregular waves at 40 knots and SSN=5.

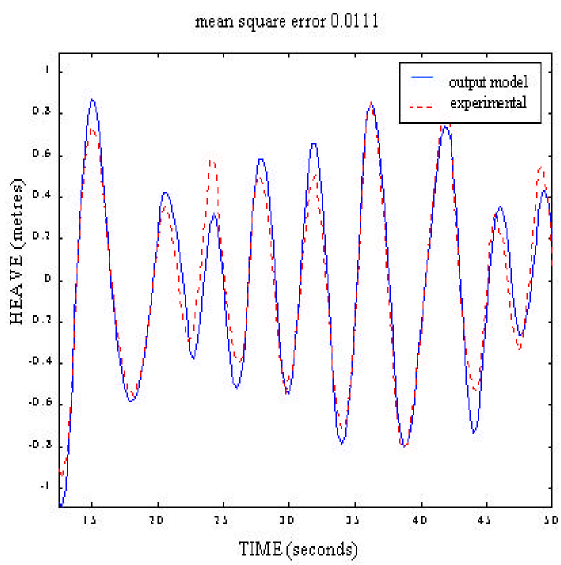

Fig. 4. Simulation of $G_H(s)$ and measured heave at 40 knots and sea state numbre (SSN) equal to 5.

Figure 5 shows the output of $G_P(s)$ and the measured pitch in the CEHIPAR when the input was irregular waves at 40 knots and SSN=5.

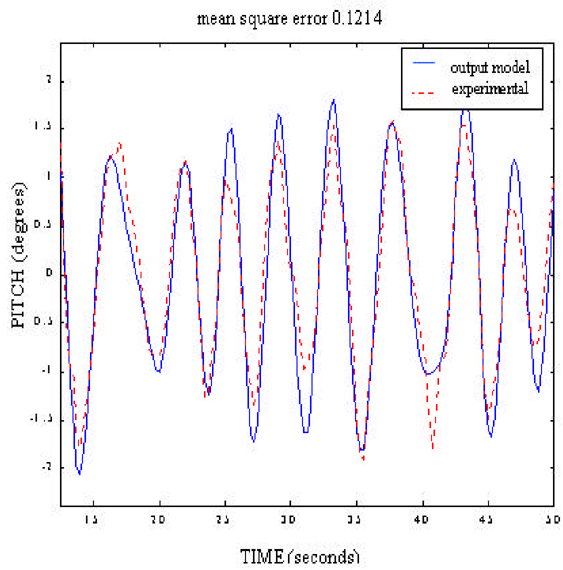

Fig. 5. Simulation of $G_P(s)$ and measured pitch at 40 knots and sea state number (SSN) equal to 5.

5. CONCLUSION

In this paper continuous linear models for vertical dynamics of a high speed ship has been showed. These models were identified by a non-linear least square algorithm applied in the frequency domain. Once the nominal model was obtained, tightest intervals around each coefficient of nominal transfer functions was created while satisfying the membership and frequency response requirements. Different model validation tests was made.

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